Cost-Risk Computations by Hand Calculator

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Abstract

To do a cost estimate properly, namely by including effects of cost-estimating uncertainty and technical and other risks, it is necessary to conduct a cost-risk analysis, the first step of which is to assign probability distributions to all WBS-element costs. In particular, a proper cost estimate for a WBS element consists of three independent components: (1) a “best” estimate cost, usually the most likely cost of the element; (2) a random variable, typically symmetric, representing uncertainty due to estimating error; and (3) a random variable, typically right-skewed, representing the cost impacts of technical and other risks. A number of types of probability distributions are used for this purpose, including the triangular, normal (Gaussian), lognormal, exponential, and uniform. For the estimating-uncertainty component, the normal distribution is often an appropriate choice, while for the risk component, the triangular appears to be the most commonly used, due primarily to its mathematical simplicity and its ability to model easily-understandable cost bounds, such as the lowest possible (most optimistic) cost, the most likely cost, and the highest possible (worst-case) cost.

After the most likely cost and the two probability distributions, along with their parameters (two for the normal, three for the triangular) are selected, the second step is to compute the statistical sum of the WBS-element cost distributions to derive the probability distribution of total cost. There are essentially three mathematical methods of carrying out this computation: (1) multidimensional calculus, specifically multiple integration for convolutions of random variables; (2) Monte Carlo sampling from the various distributions, taking account of their correlations; and (3) analytic approximation based on limit theorems of statistics. The first method is, for all practical purposes, impossible to apply, due to the extreme complexity of the integrals that would have to be worked out. To run the Monte Carlo simulations, commercial software packages, such as Crystal Ball™ and @Risk™, are available as third-party add-ons to Microsoft Excel™, but they suffer from use of Spearman correlations instead of the more appropriate Pearson correlations, as well as the usual time-related problems typical of Monte Carlo analysis. This brings us to analytic approximation.

The analytic approximation method, more precisely the “method of moments”, is faster, does correlation properly, and, using the technique discussed in this presentation, can be worked out entirely on a hand calculator. Its only possible disadvantage is that, unless your hand calculator has matrix-eigenvalue capability (which the more sophisticated ones do), the validity of the input correlation matrix has to checked offline (maybe even – sorry - on a computer!). The procedure suggested here is based on the Central Limit Theorem of statistics, which states that, if the number of WBS elements whose costs are to be summed is “large,” then the distribution of total-system cost is approximately normal (Gaussian). Furthermore, the two parameters that characterize the total-cost normal distribution are its mean, which is the sum of the means of the individual WBS-element costs, and its variance, which is the sum of the variances of the individual WBS-element costs plus a term that involves the inter-element correlations. The word “large” above really means “the larger the number of WBS elements is, the closer is the match between the actual total-cost distribution and the approximating normal distribution. Depending on the skewness of the risk components, a “good” approximation can be obtained for as few as 10 WBS elements, although inter-element correlations tend to reduce the

Contents

Contents

"Point" Cost Estimates

- **Funding Organizations Seek "Single Best Estimate" for …**
  - Cost/Performance Tradeoff Studies
  - Benefit/Cost Analyses
  - Source Selections
  - Budget Planning
- **But Program Cost is Nebulous, Heavily Impacted by …**
  - Technological (Im)maturity
  - Software Requirements
  - Programmatic Considerations
  - Schedule Slips
  - Unforeseen Events
- **While "Point" Cost Estimates are not "Correct", "Actual" Program Cost Falls within Some Range Surrounding the "Best" Estimate (with some degree of confidence)**
  - The Best We Can Hope to Do Is to Understand the Uncertainty
  - Understanding the Uncertainty Will Help Us Make Provision for It

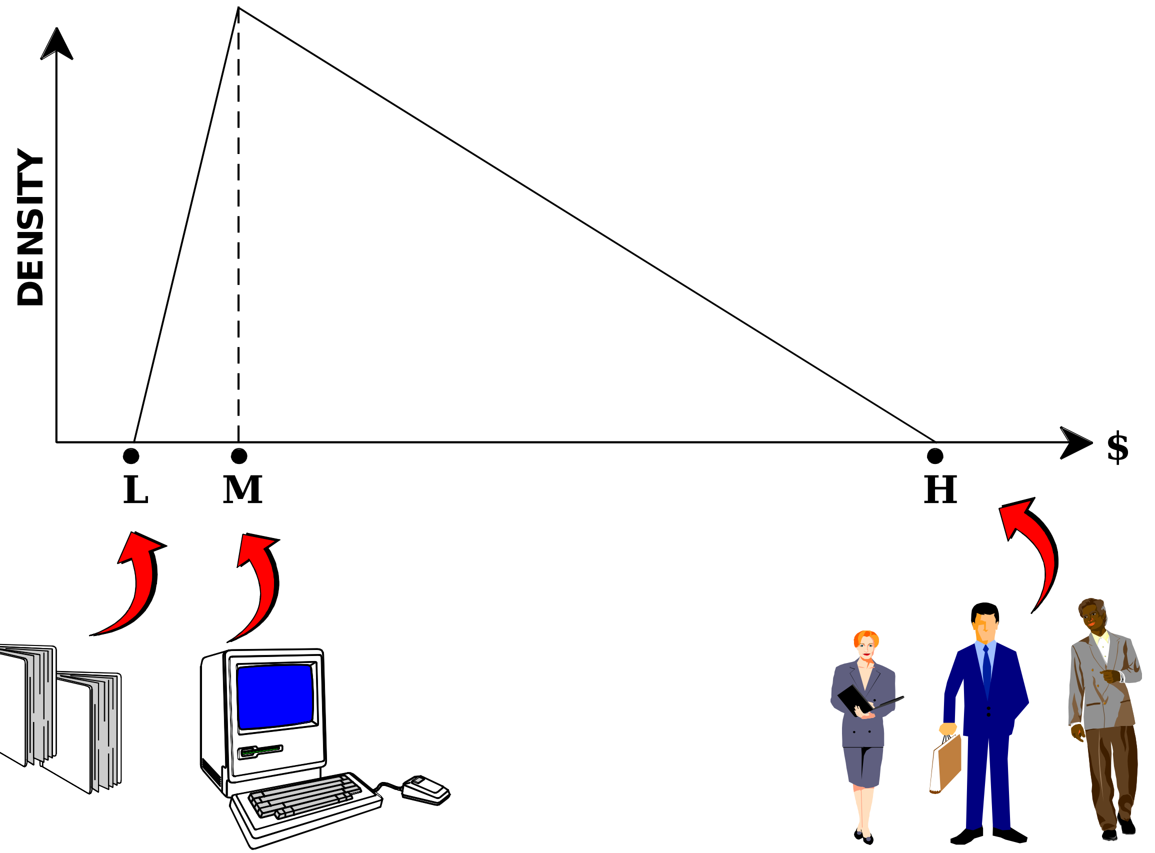
Naïve "Roll-Up" Method

- **List Cost Elements in a Work-Breakdown Structure (WBS)**
  - Calculate "Best Estimate" of Cost for Each WBS Element
  - Sum All Best Estimates
  - Define Result to be "Best Estimate" of Total-System Cost
- **Unfortunately, It Turns Out That Things are Not as Simple as They Seem: Is the "Best Estimate" …**
  - … the "Most Likely" Cost? ("Mode")
  - … the 50th-Percentile Cost? ("Median")
  - … the Expected Cost? ("Mean")
- **When Estimating Costs of Complex Systems, These Three Numbers are Almost Always Different**

Solution: Model Costs Statistically

- **While It is Being Estimated, "Actual" Program Cost is an Uncertain Quantity**
  - The "Point" or "Best" Estimate is not the Only Possible Estimate – This Means that Other Estimates are "Worse"
  - Use of Phrase "Most Likely Cost" Implicitly Assumes that Other Cost Levels are "Less Likely"
  - "Most Likely (Mode)," "50^{th} Percentile (Median)," and "Expected Value (Mean)," are Statistical Terms Characteristic of Probability Distributions
- **This Terminology Implies that Costs are Statistical in Nature and are Defined by their Probability Distributions**

Triangular Distribution of Element Cost

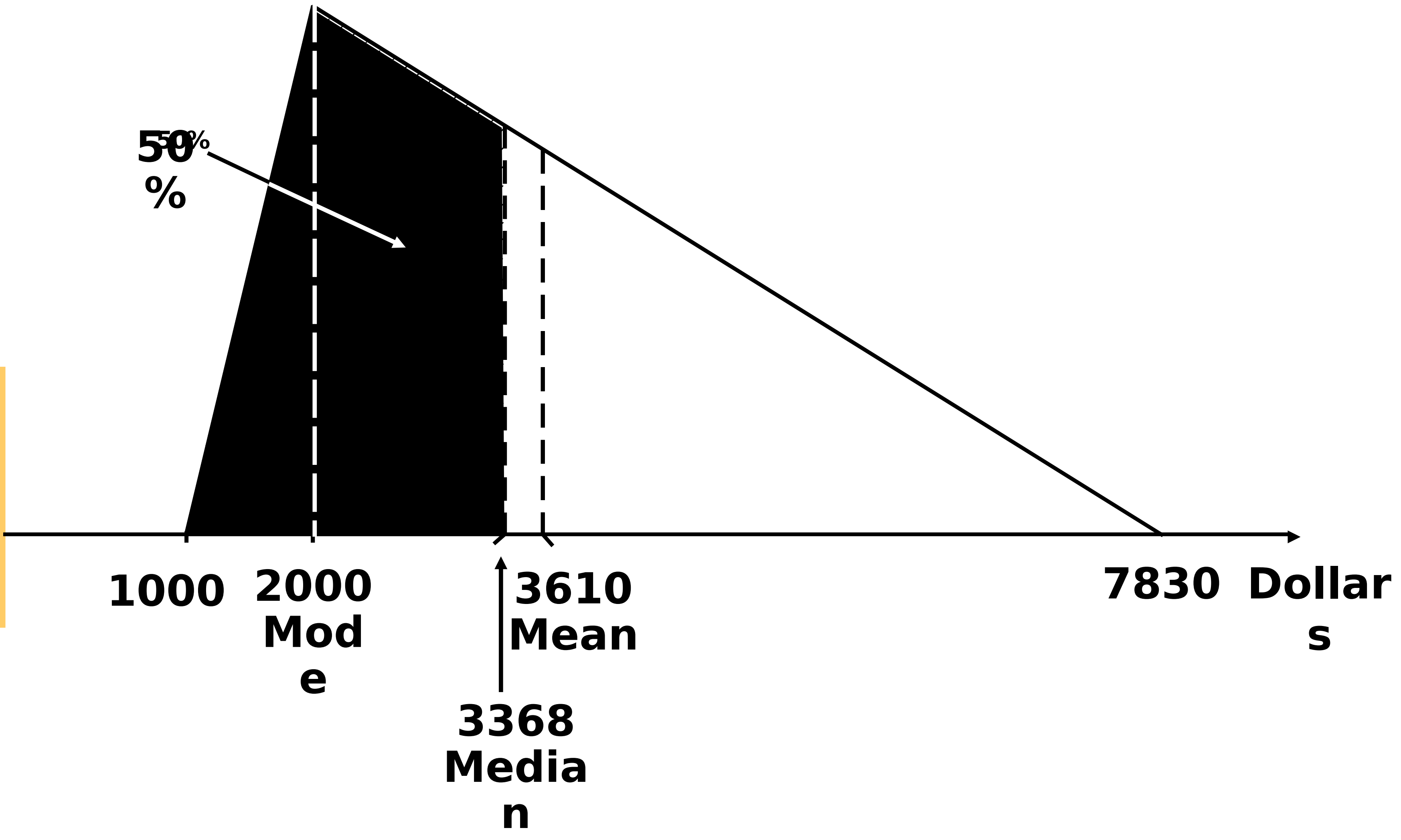


Optimistic Cost

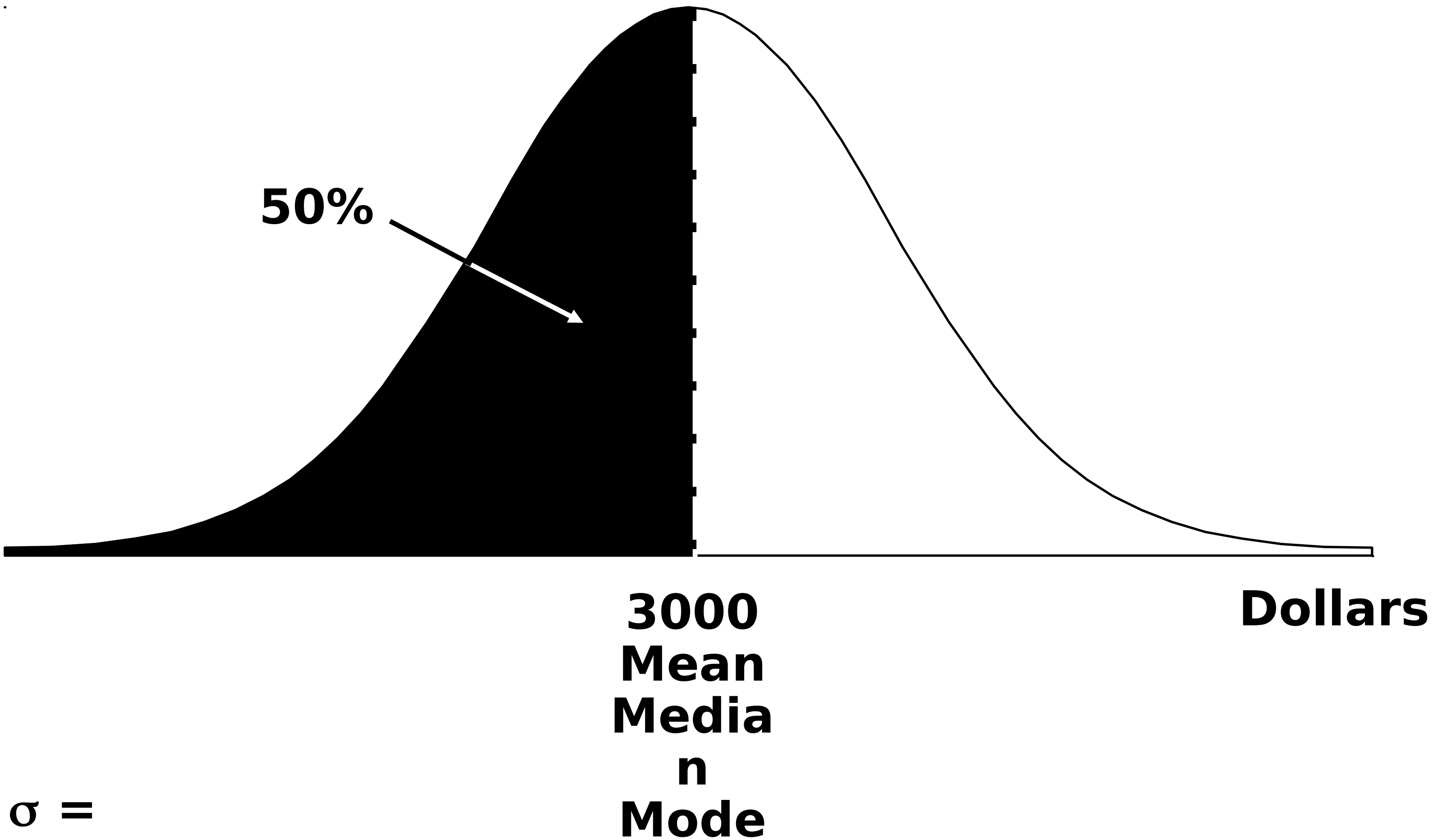
Best-Estimate Cost (Mode)

Cost Implication of Technical Assessment

Statistics of the Triangular Distribution



Statistics of the Normal (Gaussian) Distribution



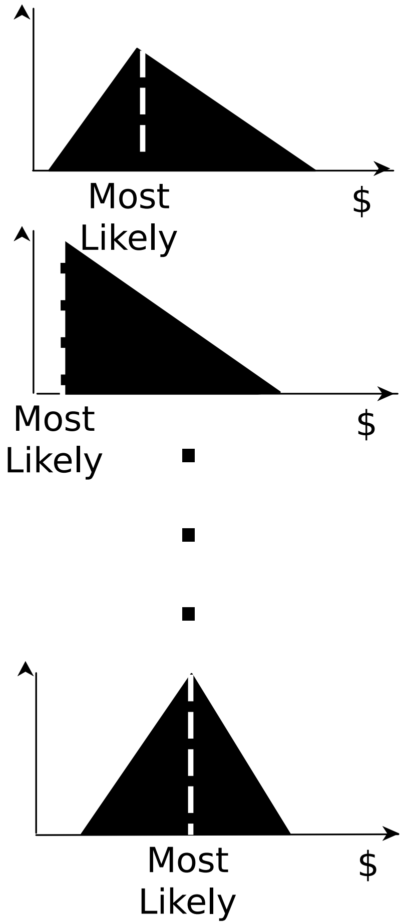
σ = 1000

Summing Element Costs Statistically

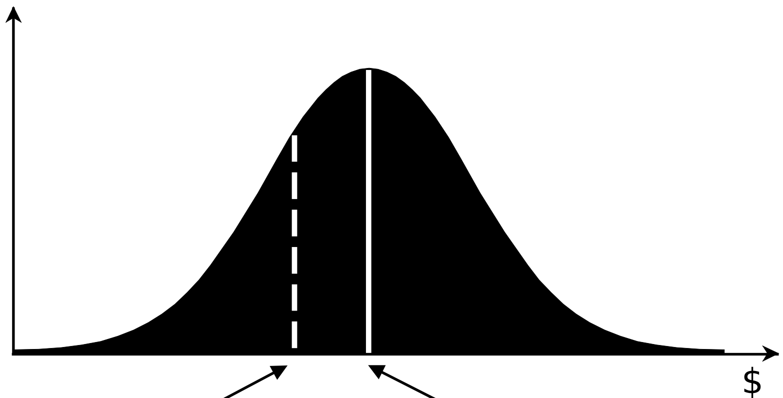
- ***Central Limit Theorem of Statistics*: If Number of WBS Elements is "Large," Distribution of Total Cost is Approximately Normal (Gaussian)**
- ***Another Statistical Theorem*: Sum of WBS-Element Means = Mean of Total-Program Cost**
- **Therefore Mean = Median = Mode for Total-Cost Distribution**
  - Total-Cost Mean = Sum of WBS-Element Means
  - Total-Cost Median = Sum of WBS-Element Means
  - Total-Cost Mode = Sum of WBS-Element Means
- **It Inexorably Follows that …**
  - Total Cost Median **>** Sum of WBS-Element Medians
  - Total Cost Mode **>** Sum of WBS-Element Modes

A Picture of the Central Limit Theorem

WBS-ELEMENT TRIANGULAR COST DISTRIBUTIONS



MERGE WBS-ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST NORMAL DISTRIBUTION



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What the Central Limit Theorem Means for Decision Makers

- **Naïve Roll-up Procedure Does Not Work**
  - Sum of WBS-Element Most Likely Costs (Modes) Does Not Equal Most Likely Total Cost (Mode)
  - Sum of WBS-Element 50th-Percentile (Median) Costs Does Not Equal the 50th-Percentile (Median) Total Cost
- **Only Statement that is True: Sum of the WBS-Element Expected (Mean) Costs Equals Expected (Mean) Total Cost**
- **Unfortunately, though, Only Percentiles Are Meaningful to Decision Makers for Budgeting and Program-Control Purposes**
  - If Budget is Set at 50th-Percentile Cost, Probably of Overrun is 50%
  - If Budget is Set at 80th-Percentile Cost, Probably of Overrun is 20%

Cost-Risk Analysis

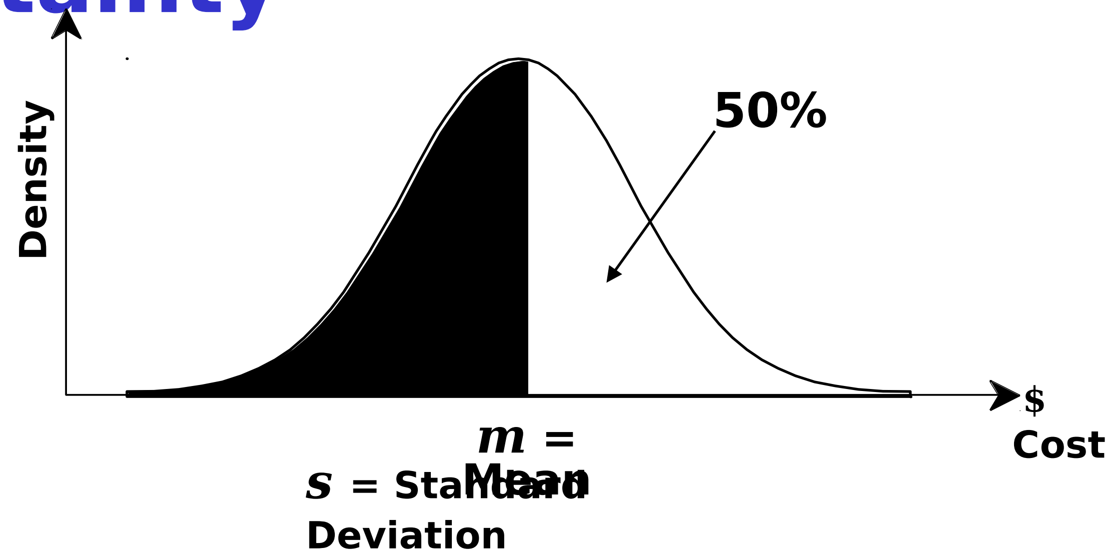
- **“Cost Risk”: A Working Definition**
  - Inadequacy of Forecasted Funding Requirements to Assure That Program Can Be Completed and Meet Its Stated Objectives
- **“Cost-Risk Analysis”: A Procedure**
  - Model WBS-element Costs As Uncertain Quantities (i.e., Random Variables) That Have Probability Distributions
  - Combine WBS-element Cost Distributions Statistically (e.g., by Monte Carlo Sampling) to Generate Cumulative Distribution of Total Program Cost
  - Read off 70th Percentile Cost, 90th Percentile Cost, etc., from Cumulative Distribution to Estimate Additional Amount of Dollars Needed to Cover Risk
  - Quantify Confidence in Anybody’s “Point” Estimate of Program Cost or in Budgeted Funding

Contents

Components of the Estimate

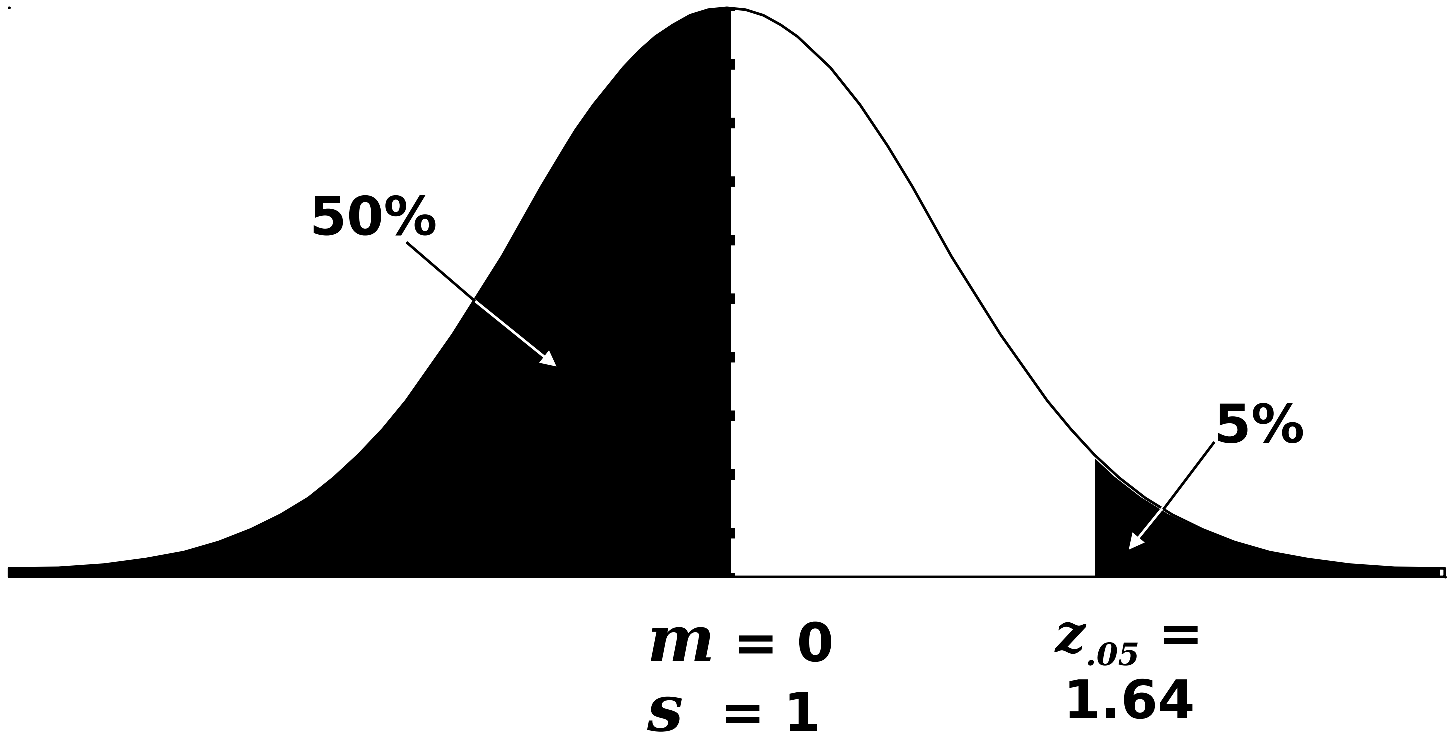
- **Best Estimate, Defined as Most Likely Cost, Derived from …**
  - Cost-Estimating Relationship (CER)
  - Analogy
  - Vendor Quote
  - Output of Commercial Parametric Model
- **Normal (Gaussian) Distribution Representing Estimating Uncertainty, Characterized by …**
  - CER Standard Error and Bias
  - Historical Record of Vendor-Quote Accuracy
  - Method Appropriate for Analogy or Model Output
- **Triangular Distribution Representing Impact of Technical, Programmatic Risks, Bounded by …**
  - Optimistic (Best-Case) Bound Usually Found in Contractor Proposal
  - Pessimistic (Worst-Case) Bound Derived from Program's Risk "Watch List" or Risk Management Plan

Normal (Gaussian) Distribution of WBS-Element Estimating Uncertainty



- Graph of Probability Density Function
- Total Area Under Gaussian Curve = 1.00
- Two Parameters m,s Completely Specify Distribution
- Mean = Median = Mode, All Percentiles can be Expressed in Terms of m and s

Standard Normal Distribution



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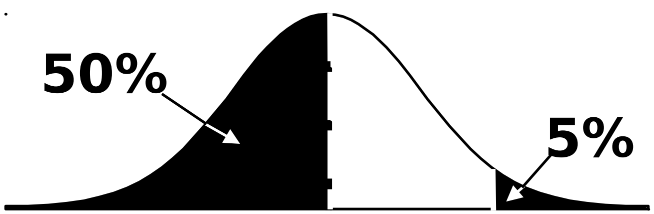
Statistical Metrics of Normal Distribution

- Mode = m
- Median = 50th Percentile = m
- Mean = m
- Standard Deviation = s

PERCENTILE	$1-\alpha$	z_α
95	.95	1.64485
90	.90	1.28155
80	.80	0.84162
70	.70	0.52440
60	.60	0.25335
50	.50	0.00000
40	.40	-0.25335
30	.30	-0.52440
20	.20	-0.84162
10	.10	-1.28155
5	.05	-1.64485



- $N_{1-\alpha}$ = Dollar Value at which P{Cost $\leq N_{1-\alpha}$} is $1-\alpha$

  = (Equivalently) Dollar Value at which P{Cost > $N_{1-\alpha}$} is α

  $= m + z_\alpha s$

  where z_α = αth Percentage Point of the Standard Normal Distribution (see previous chart and table above)

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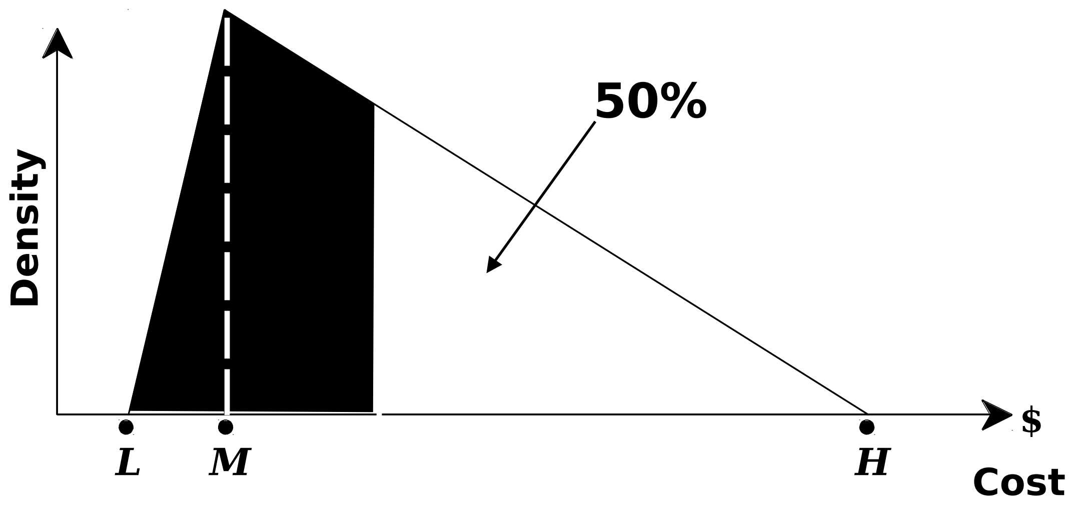
Typical Technical Risk Drivers

- **Beyond-State-of-the-Art Technology Needs**
  - Data-Processing Capability
  - Communication Links
  - Survivability
  - Power Requirements
  - Thrust
  - Cooling
- **Special Requirements**
  - Geographic Distribution of Networks
  - Security Equipment
  - Procurement Quantities
- **Software Solutions**
  - Development
  - Integration of COTS
  - Testing

Typical Programmatic Risk Drivers

- **System Integration and Testing**
  - Multicontractor Teams
  - Conflicting Schedules and Workloads
  - Government-Furnished Equipment (GFE)
- **Tight Schedules**
  - Requirements for Undeveloped Technology
  - Software Development and Testing
  - Integration of COTS Software
- **Limited Resources**
- **Program Funding Stretch-Out**
- **Security Requirements**
- **Supplier Viability**
- **Unforeseen Events**

Triangular Distribution of WBS-Element Risk Impact on Cost



- Graph of Probability Density Function
- Total Area of Triangle = 1.00
- Three Parameters L, M, H Completely Specify Distribution
- Mean, Median, Sigma, All Percentiles Can be Expressed in Terms of L, M, and H

Statistical Metrics of Triangular Distribution

- **Mode = M (most likely value of cost)**
- **Median** $T_{.50} = L + \sqrt{0.50(M-L)(H-L)}$ if $M - L \geq 0.50(H-L)$

  $= H - \sqrt{0.50(H-L)(H-M)}$ if $M - L \leq 0.50(H-L)$
- **T_p = Dollar Value at Which** $P\{\text{Cost} \leq T_p\} = p$

  $$T_p = H - \sqrt{(1-p)(H-L)(H-M)} \quad \text{if } p \geq \frac{M-L}{H-L}$$

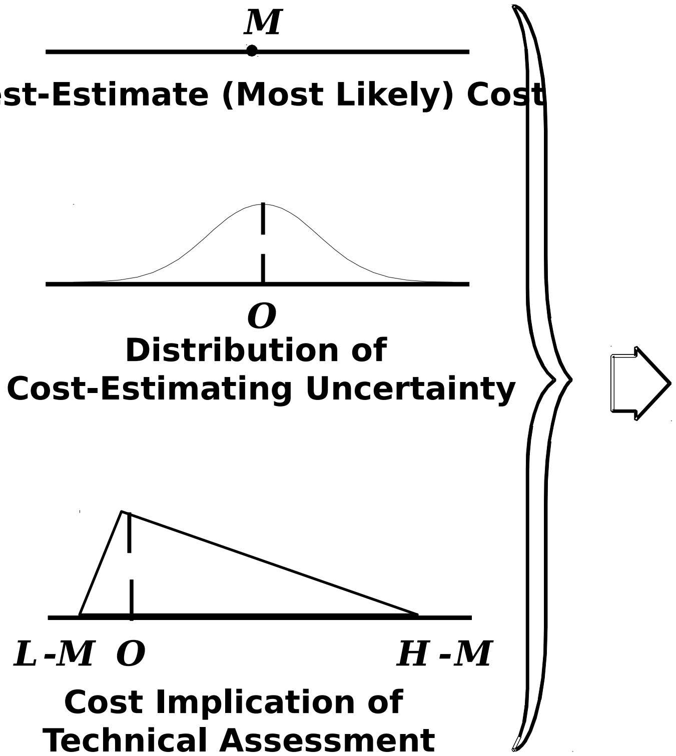
  $$T_p = L + \sqrt{p(M-L)(H-L)} \quad \text{if } p \leq \frac{M-L}{H-L}$$
- **Mean** $= \dfrac{L+M+H}{3}$

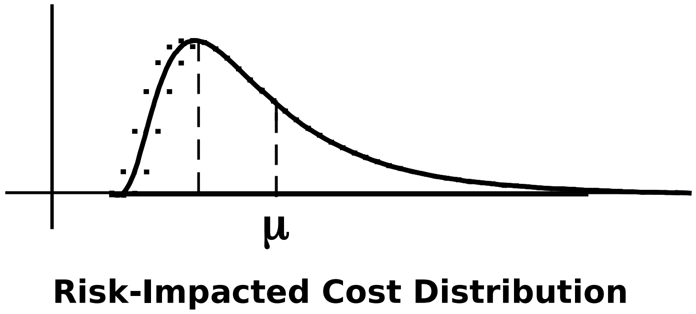
Standard Deviation $= \sigma = \sqrt{\dfrac{L^2+M^2+H^2-LM-LH-MH}{18}}$

Contents

Risk-Impacted WBS-Element Cost Distribution

Uncorrelated Components: **Summation:**





Statistical Metrics of Risk-Impacted Cost Distribution: Its Mean

- ***Statistical Theorem: Sum of Means of Random Variables = Mean of Sum of Random Variables***
- **Means of Component Distributions**
  - Mean of (Constant) Best-Estimate Cost = $\boldsymbol{M}$
  - Mean of Estimating-Uncertainty Distribution = $\boldsymbol{0}$
  - Mean of Risk-Implication Distribution = $\dfrac{(L-M)+0+(H-M)}{3}$
- **Mean of Risk-Impacted WBS-Element Distribution**

$$= \mu \quad \frac{L+M+H}{3}$$

Statistical Metrics of Risk-Impacted Cost Distribution: Its Standard Deviation

- ***Statistical Theorem: Standard Deviation of Sum of Uncorrelated Random Variables = Square Root of Sums of Squares of Standard Deviations of Random Variables***
- **Standard Deviations of Component Distributions**
  - Standard Deviation of (Constant) Best-Estimate Cost = 0
  - Standard Deviation of Estimating-Uncertainty Distribution = s
  - Standard Deviation of Risk-Implication Distribution

$$= \sqrt{\frac{(L-M)^2 + 0^2 + (H-M)^2 - (L-M)(H-M)}{18}} \qquad \sqrt{\frac{L^2 + M^2 + H^2 - LH - LM - MH}{18}}$$

- **Standard Deviation of Risk-Impacted WBS-Element Distribution**

$$\sigma = \sqrt{s^2 + \frac{L^2 + M^2 + H^2 - LH - LM - MH}{18}}$$

WBS-Element Risks are Correlated

- Resolving One Element's Risk Issues (by expensive fix or bypass) Usually Involves Making Adjustments to (thereby increasing costs of) Other Elements
  - Technical Risks in, e.g., Antenna Subsystem of a Radar System Will Likely Induce Weight (and therefore Cost) Growth in Power, Platform, and Other Subsystems, as well as Cost Growth in Software
  - Schedule Slippage Due to Problems in One Element Lead to Cost Growth in Other Elements ("standing army" effect)
  - Hardware Problems Discovered Late in Program Often Have to be Circumvented (at least in the short term) by Making Expensive Last-Minute Fixes to Software (which then show up as software cost overruns)
- Occasionally Correlations are Negative
  - Risk dollars judiciously applied in one area may reduce costs in other areas
  - Technical fix in one area may be applicable to (and lessen risk in) other areas
- Numerical Values of Inter-Element Correlations ρ_{ij} are Difficult to Estimate, but That's Another Story

Correlation Affects Standard Deviation

- $X_1, X_2, \ldots, X_n$ are Costs of WBS Elements (Random Variables)
- Total Cost $= \sum_{k=1}^{n} X_k = X_1 + X_2 + \ldots + X_n$
- Mean of Total Cost $= \mu_T = E\left[\sum_{k=1}^{n} X_k\right] = \sum_{k=1}^{n} E(X_k) = \sum_{k=1}^{n} \mu_k$
- Variance of Total Cost $= Var\left[\sum_{k=1}^{n} X_k\right]$

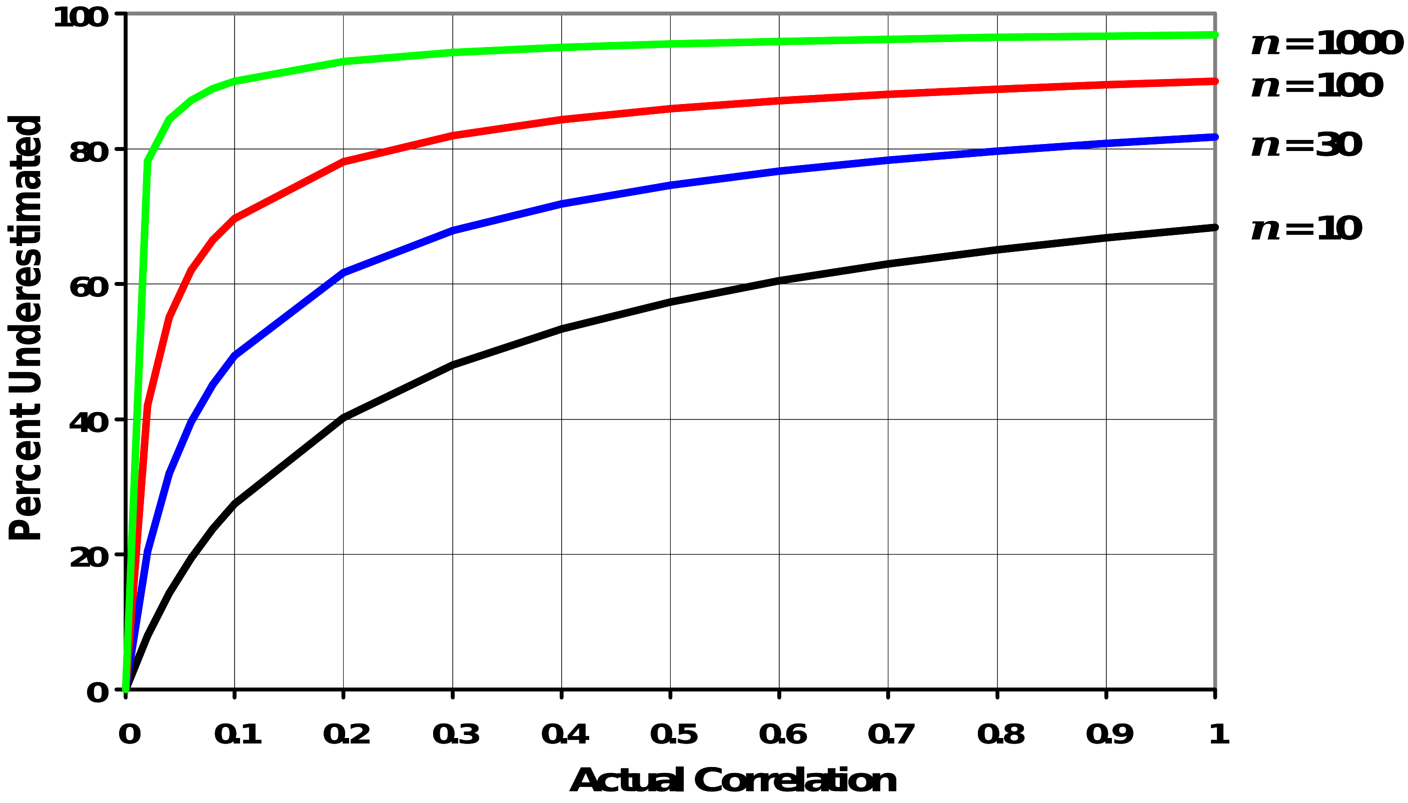
$$= \sum_{k=1}^{n} \sigma_k^2 + 2\sum_{j=2}^{n} \sum_{i=1}^{j-1} \rho_{ij}\sigma_i\sigma_j$$

- Standard Deviation of Total Cost

$$\sigma_T = \sqrt{\sum_{k=1}^{n} \sigma_k^2 + 2\sum_{j=2}^{n} \sum_{i=1}^{j-1} \rho_{ij}\sigma_i\sigma_j}$$

Why Correlation is Important

Percent by which Sigma is Underestimated in "Average" Scenario When Correlations Assumed to be 0 Instead of ρ

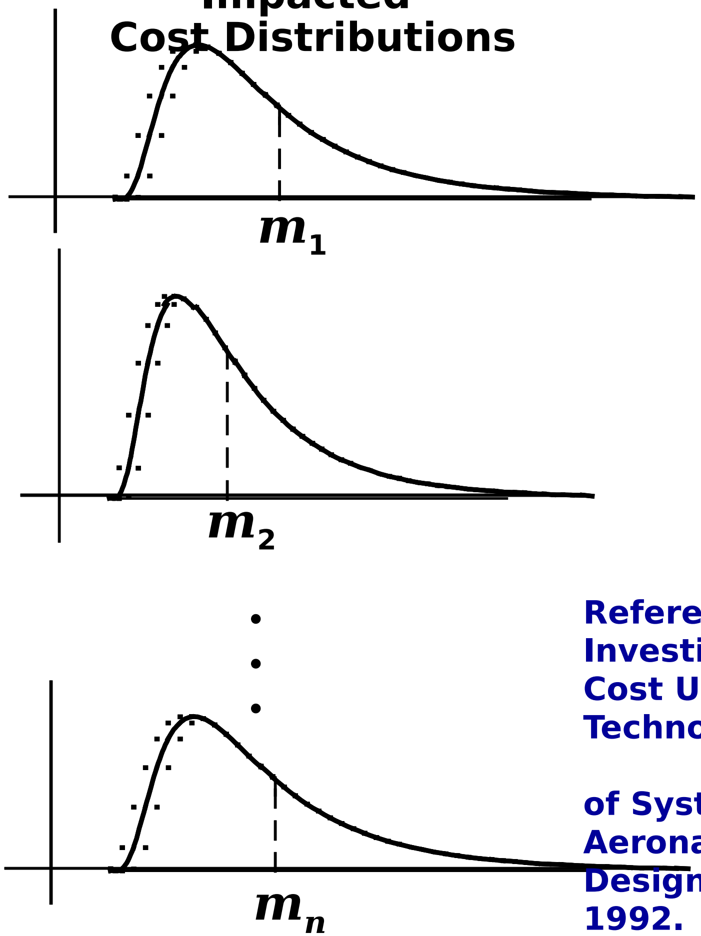


WBS-Element Cost Inputs to Total-Cost Distribution Computation

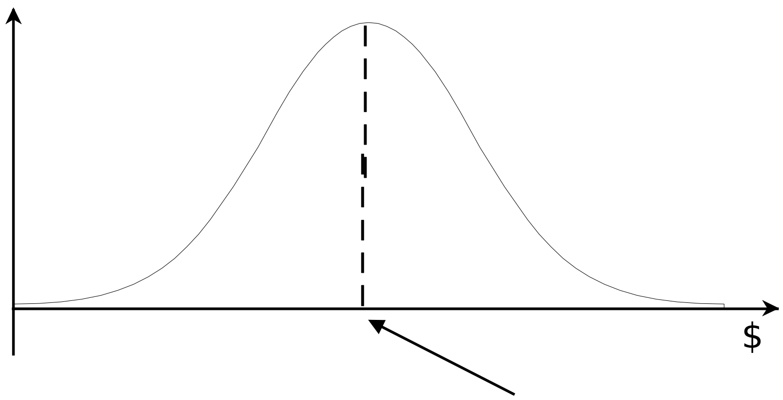
- **n = Number of WBS Elements**
- **M_j = Best Estimate (Most Likely) Cost for Element j**
- **L_j = Optimistic (Best-Case) Cost-Risk Bound for Element j**
- **H_j = Pessimistic (Worst-Case) Cost-Risk Bound for Element j**
- **s_j = Standard Deviation of Estimating Uncertainty for Element j**
- **[ρ_{ij}] = Correlation Matrix for Pairs of WBS Elements i, j**
  - All Diagonal Entries $\rho_{jj} = 1$
  - Correlation Matrix is "Symmetric", i.e., $\rho_{ij} = \rho_{ji}$
  - Correlation Matrix Must Be Nonnegative Definite, i.e., None of Its Eigenvalues May be Negative, in Order that the Set of Pairwise Correlations be Internally Consistent

What the Central Limit Theorem Guarantees (if *n* is "Large") …

WBS-Element Risk-Impacted Cost Distributions



Statistical Sum of WBS-Element Cost Distributions Equals Normal Distribution of Total Cost



References: 1. W.P. Simpson and K.P. Grant, "An Investigation of the Accuracy of Heuristic Methods for Cost Uncertainty Analysis," Air Force Institute of Technology, Report AU-AFIT/LA-TR-94-1, August 1994

2. P.H. Young, "FRISK – Formal Risk Assessment of Systems Cost Estimates," American Institute of Aeronautics and Astronautics, AIAA 1992 Aerospace Design Conference, Paper AIAA 92-1054, February 1992.

Computations: First Step

- Use M_j, L_j, H_j, s_j to Calculate Mean μ_j and Standard Deviation σ_j of Cost Distribution of WBS Element j
- Because the Mean and Standard Deviation are Called "Moments" of the Probability Distribution, the Technique We Are Using is Referred to as the "Method of Moments"
- $\mu_j = \dfrac{L_j + M_j + H_j}{3}$

$\forall$ $\sigma_j = \sqrt{s_j^2 + \dfrac{L_j^2 + M_j^2 + H_j^2 - L_jH_j - L_jM_j - M_jH_j}{18}}$

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Computations: Second Step

- **Use All Values μ_j , σ_j, ρ_{ij} to Calculate Mean μ_T and Standard Deviation σ_T of Total-Cost Distribution**

$$\forall \; \mu_T = \sum_{j=1}^{n} \mu_j$$

$$\forall \; \sigma_T = \sqrt{\sum_{j=1}^{n} \sigma_j^2 + 2\sum_{j=2}^{n} \sum_{i=1}^{j-1} \rho_{ij}\sigma_i\sigma_j}$$

- **IMPORTANT NOTE: The Specific Shapes Themselves of the Input Probability Distributions (such as triangular) are not Used in the Computations - Only their Means and Standard Deviations Impact the Total-Cost Distribution**

Contents

- **Cost-Risk Analysis**
  - The Cost-Risk Imperative
  - Cost-Element Probability Distributions
- **Mathematical Descriptors of Cost Distributions**
  - "Best Estimate" (Most Likely) Cost
  - Estimating-Uncertainty (Normal) Component
  - Risk (Triangular) Component
- **The Analytic Approximation**
  - Central Limit Theorem
  - Correlation
  - Method of Moments
- **How to Apply the Technique**
  - Calculating Percentiles of Total Cost
  - Calculating Confidence Levels of Estimates and Budgets
- **Summary and Conclusion**

Recall: Statistical Metrics of Normal Distribution

- Mode = m
- Median = 50th Percentile = m
- Mean = m
- Standard Deviation = s

PERCENTILE	$1-\alpha$	z_α
95	.95	1.64485
90	.90	1.28155
80	.80	0.84162
70	.70	0.52440
60	.60	0.25335
50	.50	0.00000
40	.40	-0.25335
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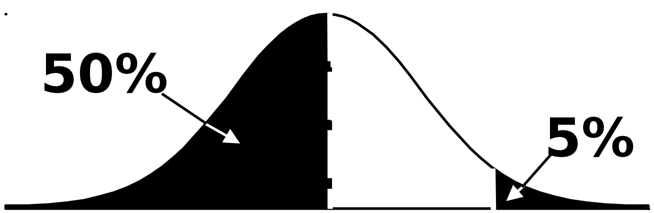
5	.05	-1.64485



- $N_{1-\alpha}$ = Dollar Value at which P{Cost $\leq N_{1-\alpha}$} is $1-\alpha$

  = (Equivalently) Dollar Value at which P{Cost > $N_{1-\alpha}$} is α

  = $m + z_\alpha s$

  where z_α = αth Percentage Point of the Standard Normal Distribution (see table above)

Computational Procedure: Third Step

- **Use μ_T , σ_T to Calculate Percentiles $N_{1-\alpha}$ of Total-Cost Distribution**

Percentile Level	$1-\alpha$	α	z_α	Cost Percentile $N_{1-\alpha}$
10th	.10	.90	-1.28155	$\mu_T - 1.28155\ \sigma_T$
20th	.20	.80	-0.84162	$\mu_T - 0.84162\ \sigma_T$
30th	.30	.70	-0.52440	$\mu_T - 0.52440\ \sigma_T$
40th	.40	.60	-0.25335	$\mu_T - 0.25335\ \sigma_T$
50th	.50	.50	0.00000	μ_T
60th	.60	.40	0.25335	$\mu_T + 0.25335\ \sigma_T$
70th	.70	.30	0.52440	$\mu_T + 0.52440\ \sigma_T$
80th	.80	.20	0.84162	$\mu_T + 0.84162\ \sigma_T$
90th	.90	.10	1.28155	+1.28155

Have You Heard This Before?

- **"Your Cost Estimate Has to be Wrong, because We Don't Have that Much Money in the Budget."**
- **When You Face this Question, Your Answer Should be…**
  - "I'm not Estimating the Cost. I'm Estimating the Probability You Can Do the Job with the Amount of Funding that's in the Budget."
  - "That's Why I Provided You a 50th-Percentile Estimate, a 70th-Percentile Estimate, and a 90th Percentile Estimate."
  - "Now, if You'll Tell Me How Much You Have in the Budget, I'll Tell You My Estimate of the Probability that the Budgeted Amount will be Sufficient to Fund the Program."

Estimating Level of Confidence Associated with Budgeted Funding

- **B = Budgeted Funding for Program**
  - A Constant
  - In Many Cases Determined by Process Other than Valid Cost Estimate
- **C = Actual Cost of Program**
  - A Random Variable
  - Distributed According to the Normal Distribution with Mean μ_T and Standard Deviation σ_T
- **We Have to Compute the Probability that the Cost C will not Exceed the Budget B, namely $P\{C \leq B\}$**

Probability that Cost C Does Not Exceed Budget B

- **Based on Statistical Properties of the Normal Distribution, We Set $P\{C \le B\}$ Up for Computation as Follows:**

$$P\{C \le B\} = P\left\{\frac{C - \mu_T}{\sigma_T} \le \frac{B - \mu_T}{\sigma_T}\right\} = P\left\{Z \le \frac{B - \mu_T}{\sigma_T}\right\}$$

- **Here $Z = \dfrac{B - \mu_T}{\sigma_T}$ is Called the "z Score" of B**

- **Z is Distributed According to the <u>Standard</u> Normal Distribution**

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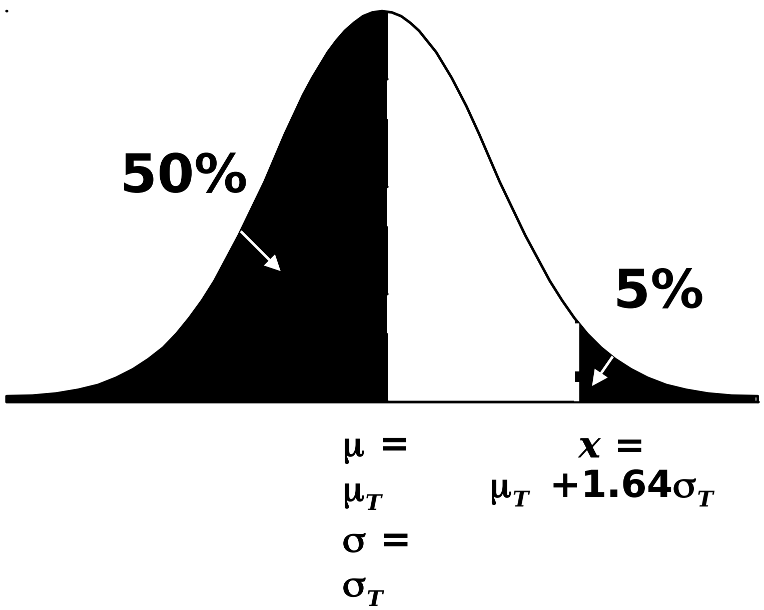
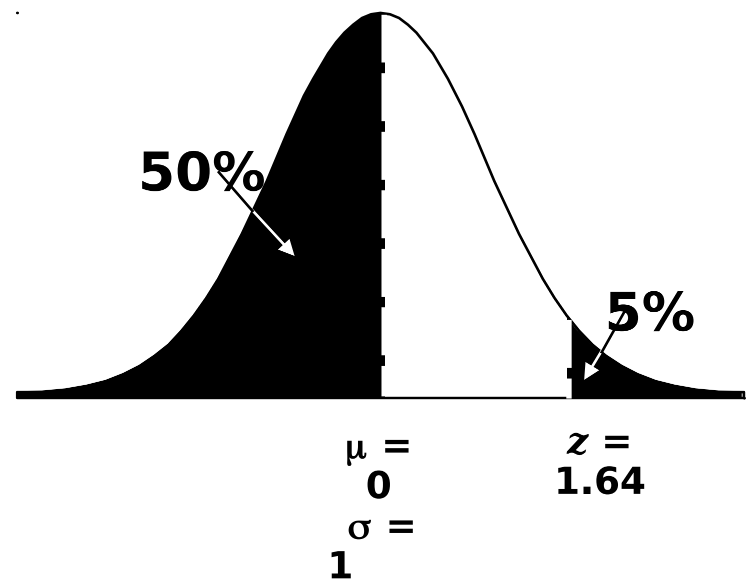
z Scores Link General and Standard Normal Distributions

$$z = \frac{x - \mu_T}{\sigma_T} = \frac{(\mu_T + 1.64\sigma_T) - \mu_T}{\sigma_T} = \frac{1.64\sigma_T}{\sigma_T} = 1.64$$

General Normal Distribution



Standard Normal Distribution



Computing the Probability that Cost C Does Not Exceed Budget B

$$P\{C \le B\} = P\left\{Z \le \frac{B-\mu_T}{\sigma_T}\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{B-\mu_T}{\sigma_T}} e^{-\frac{u^2}{2}}\, du = \Phi\left(\frac{B-\mu_T}{\sigma_T}\right),$$

The integral calculus in the above expression does not have to be worked out. The probability can be looked up in a table of the standard normal distribution (vintage 1820) or computed using a hand calculator (vintage 1990)

Table of Standard Normal Distribution

Values of

$$z = \frac{B - \mu_T}{\sigma_T}$$

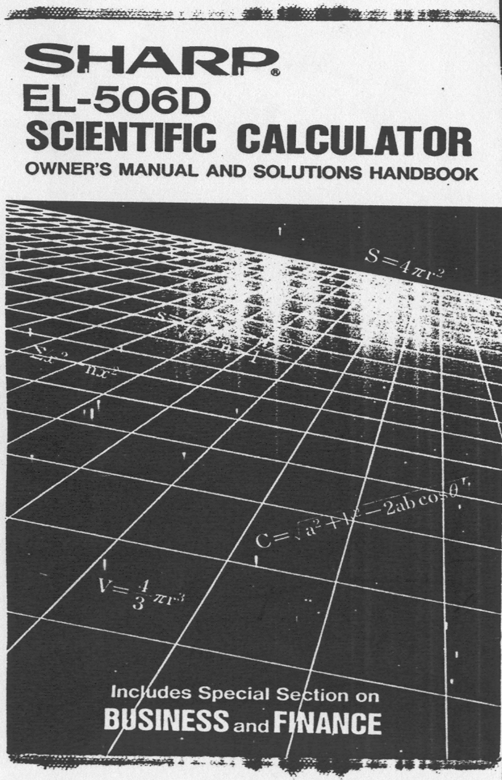
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5190	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7969	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8513	0.8554	0.8577	0.8529	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	

Values of

$$P\{C \leq B\}$$

For negative values of z, use the normal distribution's symmetry.

Normal Percentiles by Hand Calculator



[FSE] Until the FIX, SCI, and ENG indicators do not show in the display.

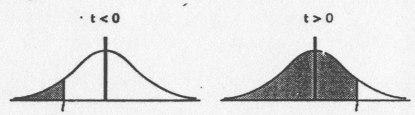
23.45 [DATA] 24.03 [DATA]
22.96 [DATA] 23.20 [DATA]
23.67 [DATA] 23.85 [DATA]
23.16 [2ndF] [t]

Result: –0.996628691 — 23.16 is within one standard deviation of the mean.

Once you have a number for t, you can apply the functions P(t), Q (t), and R(t) to that value.

THE FUNCTION P(T) AND PERCENTILES

The function P(t) represents the cumulative fraction of the standard normal distribution that is less than the value t. Thus, the value of P(t) corresponds to the area shaded in the following diagram:



Calculate P(t) by entering the value of t and pressing [2ndF] [P(t)].

Example: Calculate P(0.5).

Press: 0.5 [2ndF] [P(t)]

Result: 0.691463

A percentile is a value of P(t) that is expressed as a percent. To calculate percentiles, solve for P(t) and then multiply by 100.

page 55

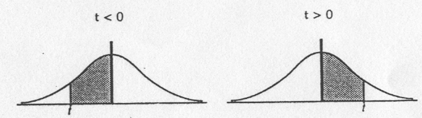
Example: What percentile is the t value 0.5?

Press: 0.5 [2ndF] [P(t)] [x] 100 [=]

Result: 69.1463

THE FUNCTION Q(T)

The function Q(t) is the cumulative fraction of the normal distribution that lies between the value t and 0. Thus, the value of Q(t) corresponds with the area shaded in the diagram below:



Calculate Q(t) by entering the value of t and pressing [2ndF] [Q(t)].

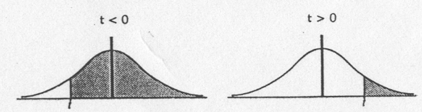
Example: Calculate Q(0.5).

Press: 0.5 [2ndF] [Q(t)]

Result: 0.191463

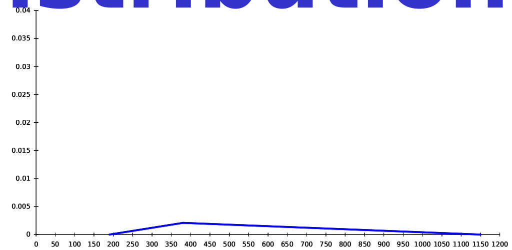
THE FUNCTION R(T)

The function R(t) is the cumulative fraction of the normal distribution that is greater than the value t. The value of R(t) corresponds with the area shaded in the diagram below.

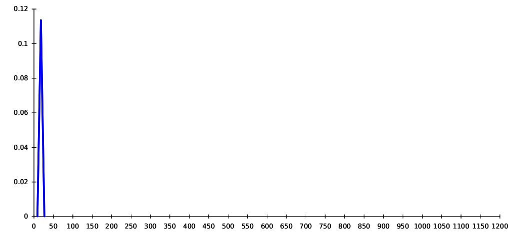


page 56 *Sharp EL-506D*

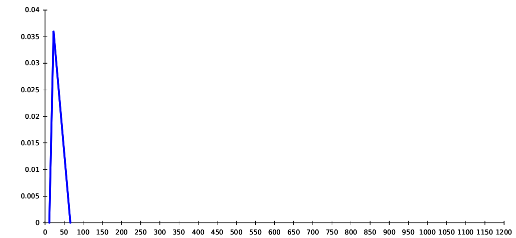
Example: System X WBS-Element Triangular Cost Distributions



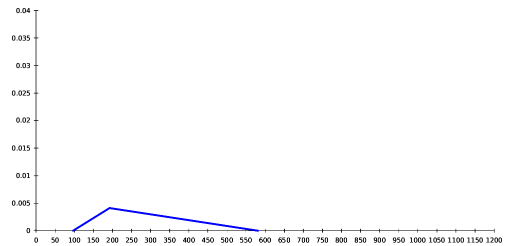
Antenna



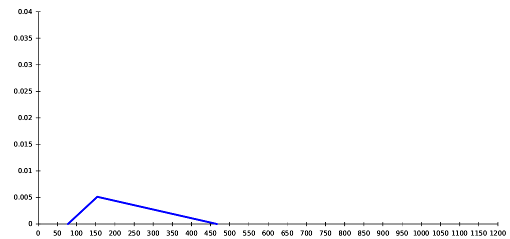
LV Adapter*



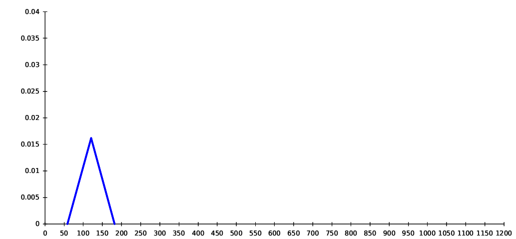
Thermal Control



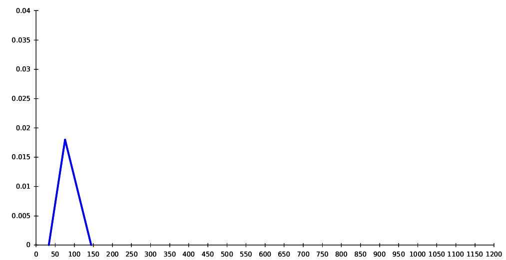
Electronics



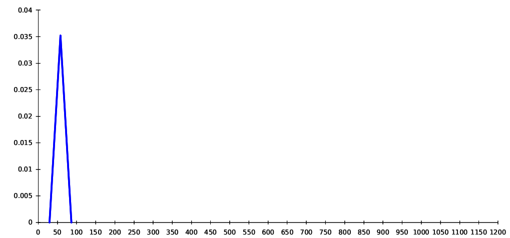
Power Distribution



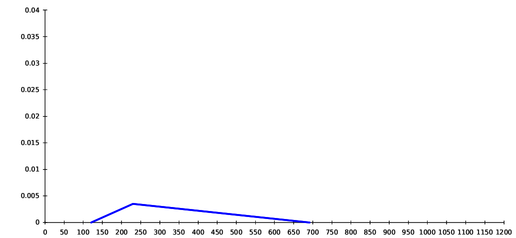
TT&C



Structure



ACS/RCS



Software

* Y scale (probability density) different in this graph only.

System X Cost Distribution Parameters

WBS Item	L	M	H	Mean	Sigma
1. Antenna	191	380	1151	574	207.62
2. Electronics	96	192	582	290	105.08
3. Structure	33	76	143	84	22.63
4. LV Adapter	9	18	27	18	3.67
5. Power Distribution	77	154	465	232	83.86
6. ACS/RCS	30	58	86	58	11.43
7. Thermal Control	11	22	66	33	11.88
8. TT&C	58	120	182	120	25.31
9. Software	120	230	691	347	123.68
SUMS				1756	279.13*

*Calculated by RSS process only, not including impact of correlations, and is not the total-cost sigma value

System X Inter-Element Correlations

Correlation Matrix

WBS Element

WBS Element	1	2	3	4	5	6	7	8	9
1	1.00	0.50	0.50	0.60	0.50	0.50	0.30	0.70	0.70
2		1.00	0.40	0.50	0.50	0.60	0.50	0.50	0.70
3			1.00	0.70	0.60	0.70	0.70	0.50	0.70
4				1.00	0.40	0.40	0.50	0.30	0.60
5					1.00	0.50	0.50	0.50	0.70
6						1.00	0.40	0.70	0.80
7							1.00	0.50	0.70
8								1.00	0.80
9									1.00

Computation of Standard Deviation of Total Cost (σ_T)

WBS Item	L	M	H	Mean	Sigma
1. Antenna	191	380	1151	574	207.62
2. Electronics	96	192	582	290	105.08
3. Structure	33	76	143	84	22.63
4. LV Adapter	9	18	27	18	3.67
5. Power Dist'n	77	154	465	232	83.86
6. ACS/RCS	30	58	86	58	11.43
7. Thermal Cont'l	11	22	66	33	11.88
8. TT&C	58	120	182	120	25.31
9. Software	120	230	691	347	123.68
SUMS				1756	279.13 *

WBS Element

WBS Element	1	2	3	4	5	6	7	8	9
1	1.00	0.50	0.50	0.60	0.50	0.50	0.30	0.70	0.70
2		1.00	0.40	0.50	0.50	0.60	0.50	0.50	0.70
3			1.00	0.70	0.60	0.70	0.70	0.50	0.70
4				1.00	0.40	0.40	0.50	0.30	0.60
5					1.00	0.50	0.50	0.50	0.70
6						1.00	0.40	0.70	0.80
7							1.00	0.50	0.70
8								1.00	0.80
9									1.00

Use the WBS-Element "Sigma" Values and the Inter-Element Correlations to Compute σ_T :

$$\sigma_T = \sqrt{\sum_{j=1}^{n} \sigma_j^2 + 2\sum_{j=2}^{n} \sum_{i=1}^{j-1} \rho_{ij}\sigma_i\sigma_j} = 491.78$$

Matrix Eigenvalues by Hand Calculator

TEXAS INSTRUMENTS

TI-85

GRAPHING CALCULATOR

GUIDEBOOK

Guidebook developed by: The staff of Texas Instruments Instructional Communications

With contributions by: Brad Christensen, Franklin Demana, Doug Feltz, Linda Ferrio, Pat Hatcher, Dave Hertling, Don LaTorre, Pat Milheron, John Powers, Dave Stone, Bert K. Waits, C. B. Wilson

Texas Instruments gratefully acknowledges the contributions of Bert K. Waits and Franklin Demana, who provided ideas for some of the applications and examples in this manual.

The MATRX MATH Menu

The MATRX MATH menu displays additional matrix math functions. Press [MORE] to move around the menu. When you select an item from the menu, the name is copied to the cursor location.

The MATRX MATH Menu — When you select ⟨MATH⟩ from the MATRX menu, the menu keys are labeled with the first five items of the menu.

det	T	norm	eigVl	eigVc
rnorm	cnorm	LU	cond	

The det Function — det (determinant) returns the determinant of a square matrix. The result is a real number if the matrix is real, a complex number if the matrix is complex.

det *matrix*

The Transpose Function — T (transpose) returns a transposed matrix. The result is a matrix in which element *row,column* is swapped with element *column,row*. For complex matrices, the result is a matrix in which element *row,column* is swapped with element *column,row*, and the conjugate is taken.

*matrix*T

The norm Function — norm returns the Frobenius norm, a number equal to $\sqrt{\Sigma(real^2+imag^2)}$ where the sum is over all elements of a real or complex matrix.

The eigVl Function — eigVl (eigenvalues) returns a list of the eigenvalues of a real or complex square matrix. The eigenvalues of a real matrix may be complex.

eigVl *matrix*

The eigVc Function — eigVc (eigenvectors) returns a matrix containing the eigenvectors for a real or complex square matrix, each column corresponding to an eigenvalue. Eigenvectors of a real matrix may be complex. Note that an eigenvector is not unique; it may be scaled by any constant factor. TI-85 eigenvectors are not normalized.

eigVc *matrix*

Eigenvalues of Correlation Matrix

- **Here They are, as Computed by my Texas Instruments TI-85 Graphing Calculator:**

5.5346
0.9146
0.7304
0.0057
0.5876
0.0910
0.4769
0.1294
0.5298

- **Because All Nine Eigenvalues are Nonnegative, the Inter-Element Correlations are Consistent**

Cost-Risk Analysis Summary of System X

Percentile	μ_T	z_α	σ_T	Cost Percentile*
10th	1756	-1.28155	491.78	1125.76
20th	1756	-0.84162	491.78	1342.11
30th	1756	-0.52440	491.78	1498.11
40th	1756	-0.25335	491.78	1631.41
50th	1756	0.00000	491.78	1756.00
60th	1756	0.25335	491.78	1880.59
70th	1756	0.52440	491.78	2013.89
80th	175	0.84162	491.78	2169.89

* Cost Percentile Approximated by $N_{1-\alpha} = \mu_T + z_\alpha \sigma_T$

Probability that Cost Will Not Exceed Budget

- Them: "If You're Telling Me that Your Cost Estimate is the 50th Percentile Value on Your Chart, Namely $1756.00M, I Can Assure You that Your Methodology is Flawed, because We Don't Have that Much Money in the Budget."
- You: "If You'll Tell Me How Much You Have in the Budget, I'll Tell You the Probability that Your Budget Will be Sufficient to Fund the Program."
- Them: "We Have Budgeted $1480.00M for This Program."
- You: "Well, then, $Z = \frac{B - \mu_T}{\sigma_T} = \frac{1480 - 1756}{491.78} = -0.561$, so According to my Sharp EL-506D Scientific Calculator, Your Probability of Being Able to Deliver the Program within Budget is only P(-0.5612) = 28.73%."

Contents

- **Cost-Risk Analysis**
  - The Cost-Risk Imperative
  - Cost-Element Probability Distributions
- **Mathematical Descriptors of Cost Distributions**
  - "Best Estimate" (Most Likely) Cost
  - Estimating-Uncertainty (Normal) Component
  - Risk (Triangular) Component
- **The Analytic Approximation**
  - Central Limit Theorem
  - Correlation
  - Method of Moments
- **How to Apply the Technique**
  - Calculating Percentiles of Total Cost
  - Calculating Confidence Levels of Estimates and Budgets
- **Summary and Conclusion**

Summary

- **Do Not Sum "Most Likely" Costs**
  – Sum of WBS-Element Most Likely Costs **NOT** Equal to Most Likely Total Cost
  – For DoD and Other High-Technology Projects, Sum of WBS-Element Most Likely Costs is Almost Certainly an Underestimate of Actual Project Cost
  – Therefore Every Cost-Analysis Job Requires a Risk Analysis
- **Costs are Random Variables, Not Deterministic Numbers**
  – System Cost not Well Represented by Any Single Possible Value of the Random Variable
  – Normal Probability Distribution Available to Serve as Simple Model for Cost-Estimating Uncertainties
  – Triangular Probability Distribution Available to Serve as Simple Model for Cost Impacts of Technical, Programmatic Risks
- **Central Limit Theorem Allows Hand-Calculator Computation of Total-Cost Probability Distribution**
  – Monte Carlo Sampling Not Required
  – Correlation Impact on Distribution Spread Correctly Modeled
  – Cost Percentiles, All Other Statistics, and Confidence Associated with Budgeted Funding or Any Other Estimate Easily Calculated

Conclusion

- **No One Should Avoid Doing Cost-Risk Analysis Because They Think …**
  - … the Mathematics is too Complicated
  - … the Mathematics Takes too Long to Work Out
  - … available Software Always Has Bugs and Never Does Exactly What I Want it to Do Anyway
  - … No One Knows What the Software is Actually Doing
  - … the Monte Carlo Process as Implemented by Standard Commercial Software Products Doesn't Model Correlation Correctly
  - …the Monte Carlo Process is Flim-flam Anyway
- **Because All the Computations Can be Done Simply by Using a Hand Calculator, They're Going to Have to Think Up Another Reason!**

Speaker's Bio

Dr. Stephen A. Book is Chief Technical Director of MCR, Inc. In that capacity, he is responsible for ensuring technical excellence of MCR products, services, and processes by encouraging process improvement, maintaining quality control, and training employees and customers in cost and schedule analysis and associated program-control disciplines. Dr. Book joined MCR in January 2001 after 21 years with The Aerospace Corporation, holding the title "Distinguished Engineer" during 1996-2000 and having served as Director, Resource and Requirements Analysis Department, during 1989-1995. He has given numerous technical and tutorial presentations on cost-risk analysis and other statistical aspects of cost and economics to DoD, NASA, and ESA Cost Symposia, the AF/NASA/ESA Space Systems Cost Analysis Group (SSCAG), the U.S. Army Conference on Applied Statistics (ACAS), and professional societies such as the International Society of Parametric Analysts (ISPA), Society for Cost Estimating and Analysis (SCEA), Military Operations Research Society (MORS), U.K. Association of Cost Engineers (ACostE), and the American Institute of Aeronautics and Astronautics (AIAA). He has served on national panels as an independent reviewer of NASA programs such as the 1997-98 Cost Assessment and Validation Task Force on the International Space Station ("Chabrow Committee") and the 1998-99 National Research Council Committee on Space Shuttle Upgrades. He currently serves as chair of the Risk Subgroup of SSCAG and is a member of the Economics Technical Committee of AIAA. Dr. Book earned his Ph.D.